Two-particle quasi-neutral kinetic model of collisionless solar wind

2003 J. Phys. A: Math. Gen. 36 6215

(http://iopscience.iop.org/0305-4470/36/22/350)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **36** (2003) 6215–6220 PII: S0305-4470(03)54956-3

Two-particle quasi-neutral kinetic model of collisionless solar wind

Yegor M Vasenin[1] **and Natalia R Minkova**[2]

[1] Department of Applied Aeromechanics, Tomsk State University, FTF, Lenin Avenue, 36, Tomsk, 634050, Russia
[2] Department of Mathematical Physics, Tomsk State University, FTF, Lenin Avenue, 36, Tomsk, 634050, Russia

E-mail: nminkova@ftf.tsu.ru and nminko@mail.tomsknet.ru

Received 17 October 2002, in final form 11 February 2003
Published 22 May 2003
Online at stacks.iop.org/JPhysA/36/6215

Abstract
We present a kinetic model of the stationary solar wind based on the equation for two-particle (electron–proton) velocity distribution. The influence of collisions and the magnetic field is neglected. The approximation of the quasi-neutral plasma is applied to close the model, which results in excluding a Coulomb potential of plasma polarization field from consideration. The kinetic equation is solved using the method of characteristics. Solar wind density and speed are evaluated by integrating the two-particle distribution function over its domain of definition formed by related characteristics. The obtained dependences of the density and velocity on heliocentric distance agree with observational data of the in-ecliptic slow solar wind.

PACS numbers: 52.25.Dg, 96.50.Ci

1. Introduction

Forty years ago, the model of Parker [1] introduced the contemporary theory of solar wind as a plasma flow ejected by the Sun. Most solar wind models are based on the continuum mechanics approach [2–5]. Some use the kinetic concept to derive equations for mean parameters of particle flow. The development of the kinetic approach to solar wind modelling has been presented in some recent publications [9–15]. Most of these investigations were triggered largely by the problem of exploring energy sources necessary for providing solar wind acceleration. Significant successes have been achieved in the past few years in the observational and theoretical study of the coronal heating and acceleration mechanisms, see, for example, [5–8]. The discussion of the thermal motion of plasma particles as an energy source of the solar wind has been renewed in recent publications presenting collisionless kinetic models [9–11]. These are based on one-particle distribution functions and assumptions allowed the calculation of the appropriate polarization potential [17]. In this paper, we develop the collisionless two-particle kinetic model [15] which results in more consistent consideration of Coulomb interaction between plasma particles and yields the closed 'neutral' problem independent of the plasma polarization potential. The kinetic equation is solved analytically by the method of characteristics. The deduced analytical dependences for mean parameters (density and speed) agree with the observational data of the in-ecliptic slow solar wind.

2. Two-particle kinetic model

The solar wind is considered as a stationary flow with spherical symmetry that is formed by the fully ionized hydrogen plasma ejected by the Sun. The influence of the magnetic field and the Sun's rotation is not taken into account. The collisions of particles are neglected in plasma where the long-range Coulomb interaction is dominant.

The related kinetic equations for the distribution functions of electron (f_e) and proton (f_p) velocities yield the following equation for the two-particle distribution function f

$$\frac{\partial f}{\partial r} + F_{re} \cdot \frac{\partial f}{\partial \varepsilon_e} + F_{rp} \cdot \frac{\partial f}{\partial \varepsilon_p} - \frac{u_{\perp e}}{r} \cdot \frac{\partial f}{\partial u_{\perp e}} - \frac{u_{\perp p}}{r} \cdot \frac{\partial f}{\partial u_{\perp p}} = 0 \tag{1}$$

if statistical independence of f_e and f_p is assumed: $f = f_e f_p$. Here $\varepsilon_e = m_e\left(u_{re}^2 + u_{\perp e}^2\right)/2$, $\varepsilon_p = m_p\left(u_{rp}^2 + u_{\perp p}^2\right)/2$; $u_r, u_\perp$ are the radial and tangential components of particle velocity, r is heliocentric distance, and m is particle mass. The indices e and p refer to electrons and protons, respectively. The forces applied to a particle in gravitational and electrostatic fields are expressed through their potentials φ and ψ, $F_{re} = -m_e \mathrm{d}\varphi/\mathrm{d}r - e\,\mathrm{d}\psi/\mathrm{d}r$, $F_{rp} = -m_p \mathrm{d}\varphi/\mathrm{d}r + e\,\mathrm{d}\psi/\mathrm{d}r$, where e is the charge of an electron (proton), $\varphi = -\gamma M/r$, γ is the gravitational constant, and M is the mass of the Sun. The Coulomb potential $\psi(r)$ is produced by plasma polarization. We assume that it is a statistically averaged function not depending on the individual interactions of particles (collisions).

Let us consider the plasma flow whose typical scale length L is of the order of current heliocentric distance r. In this case an approximation of quasi-neutral plasma can be applied because $r \gg a_e$, where a_e is plasma polarization length (Debye length) which is estimated to be about dozens of centimetres in the solar corona and about dozens of metres at a distance corresponding to the Earth's orbit. This means the electron and proton densities (N_e, N_p) and their flux densities (Nu_e, Nu_p) are assumed to be equal at any heliocentric distances:

$$N_e(r) = N_p(r) \qquad Nu_e(r) = Nu_p(r). \tag{2}$$

Therefore the following integrals of the two-particle distribution function f can be interpreted as squares of plasma density N and its flux density Nu, respectively

$$N^2(r) = \int_D f(r, \vec{U})\,\mathrm{d}\vec{U} \qquad (Nu(r))^2 = \int_D u_{re} u_{rp} f(r, \vec{U})\,\mathrm{d}\vec{U} \tag{3}$$

where $\vec{U}$ is a set of variables $u_{re}, u_{\perp e}, u_{rp}$, $\mathrm{d}\vec{U} = 2\pi u_{\perp e}\,\mathrm{d}u_{\perp e}\,\mathrm{d}u_{re} 2\pi u_{\perp p}\,\mathrm{d}u_{\perp p}\,\mathrm{d}u_{rp}$.

A quasi-neutral model does not depend on the parameters of the polarization field if initial and boundary conditions are also neutral. The assumption that at the exobase $r = r_0$ the plasma state is in equilibrium yields an initial two-particle distribution f_0 as a product of Maxwell electron and proton distributions

$$f_0 = f(r_0, u_{rp0}, u_{\perp p0}) = N_0^2 \frac{(m_e m_p)^{3/2}}{(2\pi k T_0)^3} \exp\left(-\frac{\varepsilon_0}{k T_0}\right) \tag{4}$$

which satisfies the above requirement. Here $\varepsilon_0 = \varepsilon_{e0} + \varepsilon_{p0}$, k is the Boltzmann constant, T_0 is the electron and proton temperature at the exobase. The zero index marks values of variables at $r = r_0$. A solution of the kinetic equation has a domain of definition with related boundaries formed by the characteristics of this equation in the phase space of velocities. If we assume that f (including its boundary values) depends on the sum of electron and proton kinetic energies $f = f(r, \varepsilon_r, \varepsilon_\perp)$ then equation (1) can be reduced to the following form

$$\frac{\partial f}{\partial r} + (F_{re} + F_{rp}) \cdot \frac{\partial f}{\partial \varepsilon} - \frac{2\varepsilon_\perp}{r} \cdot \frac{\partial f}{\partial \varepsilon_\perp} = 0 \tag{5}$$

where

$$\varepsilon_r = \left(m_e u_{re}^2 + m_p u_{rp}^2\right)/2 \qquad \varepsilon_\perp = \left(m_e u_{\perp e}^2 + m_p u_{\perp p}^2\right)/2$$
$$\varepsilon = \varepsilon_e + \varepsilon_p = \varepsilon_r + \varepsilon_\perp \qquad F_{re} + F_{rp} = -(m_e + m_p)\,\mathrm{d}\varphi/\mathrm{d}r.$$

Thus we obtain the closed model (2), (4) and (5) which describes a stationary spherical symmetric flow of quasi-neutral plasma ejected by the Sun and driven by thermal motion of high-speed electrons [17]. By taking equation (2) into account, transformation (1)–(5) can be interpreted as a transition from the description of electron and proton statistics separately to a description of the statistics of dynamic electron–proton pairs with energy ε.

3. Solution

The first-order partial differential equation (5) resulted in $f = \text{const}$ along the characteristics:

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}r} = F_{re} + F_{rp} \qquad \frac{\mathrm{d}\varepsilon_\perp}{\mathrm{d}r} = -\frac{2\varepsilon_\perp}{r}. \tag{6}$$

The general solution to equation (5) is the arbitrary differentiable function of the first integrals E, M of the characteristic equations (6) that express the laws of energy and moments of momentum conservation

$$f = f(E, M_e, M_p) \qquad E = \varepsilon + m\varphi = \varepsilon_0 + m\varphi_0 \qquad M = r^2\varepsilon_\perp = r^2\varepsilon_{\perp 0} \tag{7}$$

where $m = m_e + m_p$. The particular integral of problem (5) and (4) is the result of substitution ε_0 from equation of energy conservation (7) in the initial distribution (4):

$$f = N_0^2 \cdot \frac{(m_e m_p)^{3/2}}{(2\pi k T_0)^3} \exp\left(-\frac{\varepsilon - m(\varphi_0 - \varphi)}{kT_0}\right). \tag{8}$$

The solution (8) is defined in the domain formed by characteristics (6) that refer to particles ejected by the Sun, i.e. to those coming from the initial domain of definition:

$$0 \leqslant u_{\perp e0} < \infty \qquad 0 \leqslant u_{re0} < \infty \qquad 0 \leqslant u_{\perp p0} < \infty \qquad 0 \leqslant u_{rp0} < \infty. \tag{9}$$

Figure 1(a) shows these characteristics in the space of u_r, $u_\perp$, $\bar{r}$, where the axis $\bar{r} = r/r_0$ starts from the initial value $\bar{r} = 1$. Then equation (9) gives the initial domain D_0:

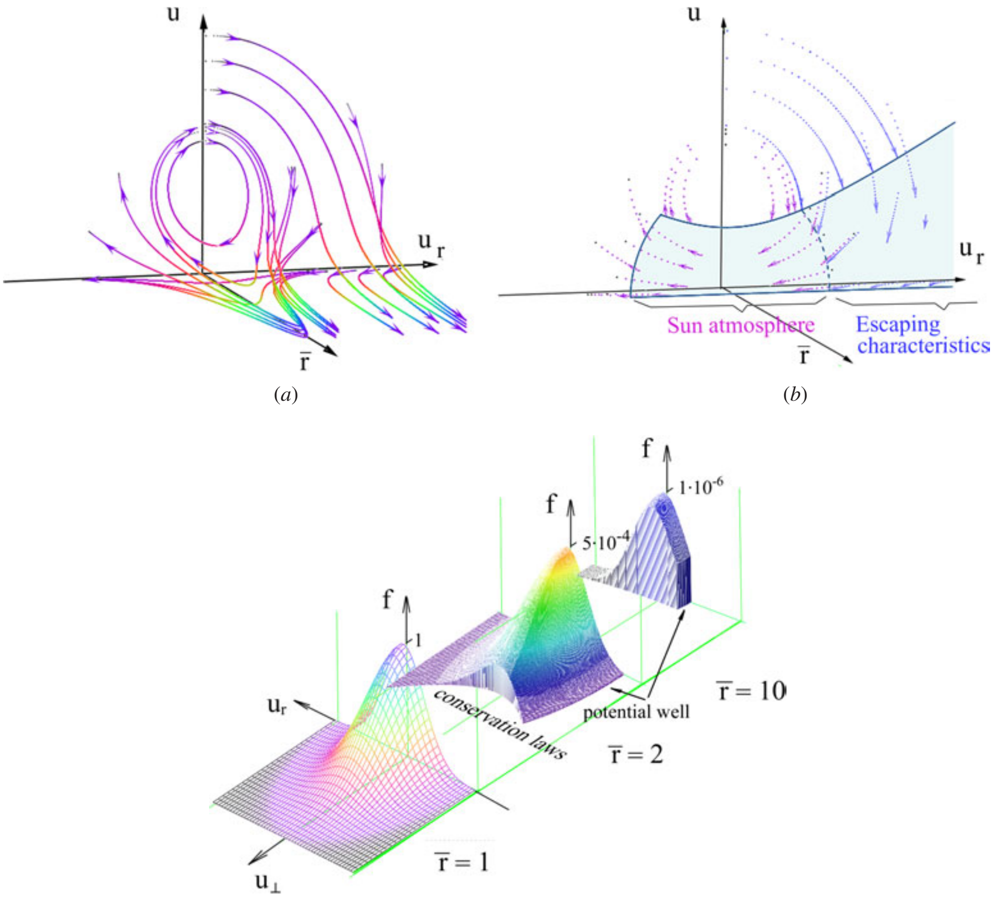
$$u_{r0} \geqslant 0 \qquad u_{\perp 0} \geqslant 0. \tag{10}$$

Here $u_r^2 = 2\varepsilon_r/m$, $u_\perp^2 = 2\varepsilon_\perp/m$. The arrows mark the direction of propagation of initial conditions along characteristics. The curves which return to the plane $\bar{r} = 1$ refer to particles of the Sun atmosphere; they do not have enough kinetic energy to overcome the gravitational well and fall to the Sun. Other high-energy particles escape the potential well and move to infinity. Thus, the domain of definition D for the solution $f(\bar{r})$ (8) at $\bar{r} > 1$ results from relations (7) and (9) and is described by the following inequalities

$$0 \leqslant u_\perp \leqslant \hat{u}_\perp \qquad -\breve{u}_r \leqslant u_r \leqslant \infty \tag{11}$$

where

$$\hat{u}_\perp = \sqrt{\frac{2}{m}\frac{\varepsilon_r + m(\varphi - \varphi_0)}{\bar{r}^2 - 1}} \qquad \breve{u}_r = \sqrt{\frac{2}{m}(-m\varphi - \varepsilon_\perp)} \qquad \bar{r} = \frac{r}{r_0}.$$



(*c*)

Figure 1. (*a*) Characteristics (9) coming from the initial domain (13) at $\bar{r} = 1$. (*b*) Domain of definition of solution (11), (14) at $\bar{r} > 1$. (*c*) Evolution of the two-particle distribution *f* along the heliocentric distance axis (scales are relative).

Figure 1(*b*) shows D at $\bar{r} > 1$ as a section of the characteristic set (it is bordered by the full curve and shaded grey).

Thus the solution of problem (5), (4) and (10) is described by function (8) and defined in domain (11).

Let us compare the considered model with the model based on equation (1) [15], whose solution is distribution (8) defined in the domain D_ψ dependent on the polarization potential ψ. It can be shown that $D_\psi \subset D$ for any admissible realization of $\psi(r)$. Therefore $N(r)$ obtained from equations (3), (8) and (10) majorizes any specific density $N_\psi(r)$ of the model [15]. It can be shown that the same relation binds the fluxes Nu_ψ and their majorant Nu evaluated by equation (3) over the domains D_ψ and D, respectively

$$Nu_\psi(r) < Nu(r). \qquad (12)$$

The evolution of both solutions $N(r)$ (8) and (11) and $N_\psi(r)$ can be illustrated by figure 1(*c*) [15]. It is seen that non-equilibrium character of the velocity distribution function intensifies as the heliocentric distance increases. This is caused by the atmosphere particles falling out as well as by energy exchange from tangential to radial motion of particles.

4. Solar wind density and speed

The obtained solution (11) and (13) of the considered model and formulae (6) yield the following evaluations of plasma flow density N and speed u

$$N(\bar{r}) = N_0 \exp\left(\frac{\bar{\varphi}_0}{2}\right)\sqrt{\eta(\bar{r})} \qquad u(\bar{r}) = \bar{u}_{0ep}\sqrt{\frac{2+(2-\bar{\varphi}_0)^2}{2\eta(\bar{r})}}\frac{1}{\bar{r}^2}$$
$$\eta(\bar{r}) = 2\exp(-\bar{\varphi})\left(1-\exp\left(\frac{\bar{\varphi}_0-\bar{\varphi}}{\bar{r}^2-1}\right)\left(1-\frac{\bar{\varphi}_0-\bar{\varphi}}{\bar{r}^2}-\frac{1}{\bar{r}^4}\right)\right)-\frac{1+(1-\bar{\varphi}_0)^2}{2\bar{r}^4} \tag{13}$$

where $\bar{\varphi} = \varphi/kT_0, \bar{u}_{0ep} = \sqrt{2kT_0/\pi\sqrt{m_e m_p}}$. Here u is evaluated as $u = Nu_\infty(\bar{r})/N(\bar{r})$ where$Nu_\infty = N_0\bar{u}_{0ep}\exp(\bar{\varphi}_0/2)\sqrt{1+2(1-(\bar{\varphi}_0/2))}\bar{r}^{-2}$ is the dominant term of descending power series of $Nu(r)$ (6) for $\bar{r} \gg 1$. Nu_∞ conserves the majorant character of Nu with respect to flux densities Nu_ψ of the model [15]. By taking into account that for the considered stationary problem $4\pi\bar{r}^2 Nu_\psi(r) = \text{const}$, equation (12) yields $Nu_\psi(\bar{r}) < Nu_\infty(\bar{r})\ \forall\bar{r} > 1$. At the same time, for the considered problem, Nu_∞ represents the flux of particles because their reverse motion from infinity is missing.

The analysis of density $N_\psi(r)$ and speed $u_\psi(r)$ as dependences on the polarization potential ψ obtained in [7] shows that they satisfy the following inequalities

$$N_\psi(\bar{r}) \leqslant N_*(\bar{r}) < N(\bar{r}) \qquad u_*(\bar{r}) \leqslant u_\psi(\bar{r}) < u(\bar{r}) \tag{14}$$

in the domain of real values of N_ψ and u_ψ : $m\varphi(2\bar{r}-1)/2e\bar{r} \leqslant \psi \leqslant m\varphi/2e\bar{r}$. Here $N_* = N_\psi$ and $u_* = u_\psi$ for $e\psi \cong m\varphi/2$ which corresponds to an approximation of quasi-neutral thermodynamic equilibrium plasma in gravitational field [16]. The above limits of admissible realizations of potential ψ restrict the existence domain of stationary solutions.

The theoretical results of the considered stationary model (13) are compared below to the empirical radial dependences deduced in [18] by averaging parameters of the in-ecliptic slow solar wind [18, 19]. This type of wind is non-steady but its mean parameters are rather stable. To compare the statistical averages (17) to these empirical data we need to assume the ergodic property of the flow.

The present model is proved in the ecliptic for heliocentric distances greater than about $6R$ where the influence of collisions can be neglected [17] (R is the Sun's radius). At the same time, the Maxwell (equilibrium) approximation of the velocity distribution at the exobase is appropriate in the collision-dominated region, i.e. at $r_0 < 6R$. Therefore we choose the exobase distance $r_0 = 5R$ and take the respective initial values of temperature and density from [18]: $T_0 \approx 8.9\times10^5$ K, $N(1) \approx 4.5\times10^{10}$ m^{-3}. In this case the difference between theoretical results (13) and empirical data [18] at $r > r_0$ is less than about 30% for density N and less than 16% for the flow speed u. The respective dependence (13) and empirical speed $u_{[18]}$ [18] satisfy the inequalities (14): $u_*(r) < u_{[18]}(r) < u(r)$ over the interval $30R < r < 370R$. At the altitude of the Earth's orbit (1 au), $u_{[18]} \approx 423$ km s^{-1}, $u \approx 483$ km s^{-1}, $N_{[18]} = 6.23\times10^6$ m^{-3}, $N = 6.25\times10^6$ m^{-3}.

If we shift the exobase to lower altitudes the Maxwellian approximation of initial distribution improves while the role of collisions increases. For $r_0 = 1.5R$ the difference between theoretical (13) and empirical [18, 19] density values does not exceed about 40% if $T_0 = 1.09\times10^6$ K and $N(1) \approx 1.8\times10^{13}$ m^{-3}. The respective profile $u(\bar{r})$ (13) leaves in the area of the observed speed's dispersion [20] but it increases more gradually at small heliocentric distances than the empirical approximation [18] and reaches the greater terminal value (at 1 au $u \approx 487$ km s^{-1}).

5. Conclusions

The suggested kinetic model is based on the two-particle distribution function and quasi-neutral plasma approximation. It allows us to evaluate the non-equilibrium distribution function and related mean flow parameters that agree in the ecliptic with the observational data for a large range of heliocentric distances. This model is independent of the parameters of the plasma polarization field and does not require an additional assumption for the Coulomb potential. As result, it includes no matching parameters and the observational data are used only for setting initial values of density and temperature.

Agreement of the theoretical results presented in this paper with the available observational data shows that the collisionless approximation is appropriate for describing mean parameters (density and speed) of the slow solar wind beyond about five Sun radii.

Acknowledgments

We are grateful to SCCS Local Organizing Committee for their financial support and hospitality that has allowed one of the authors to participate at the conference and present this paper. The work was partially supported by the Russian Foundation for Basic Research (RFBR). We would also like to thank Dr A Shamin for his consultations.

References

[1] Parker E N 1958 *Astrophys. J.* **128** 664–75
[2] Hundhausen A J 1976 *Coronal Expansion and Solar Wind* (Moscow: Mir)
[3] Baranov V B and Krasnobaev K V 1977 *Hydrodynamic Theory of Space Plasma* (Moscow: Nauka)
[4] Hollweg J V 1978 *Rev. Geophys. Space Phys.* **16** 689–720
[5] Cranmer S R 2002 *Space Sci. Rev.* **101** 229–94
[6] Fisher R and Guhathakurta M 1995 *Astrophys. J.* **447** L139–42
[7] Kohl J L *et al* 1998 *Astrophys. J.* **501** L127–31
[8] Hu Y Q, Esser R and Habbal S R 2000 *J. Geophys. Res.* A **105** 5093–111
[9] Meyer-Vernet N 1999 *Eur. J. Phys.* **20** 167–76
[10] Issautier K, Meyer-Vernet N, Pierrard V and Lemaire J 2001 *Astrophys. Space Sci.* **277** 189–93
[11] Maksimovic M, Pierrard V and Lemaire J 2001 *Astrophys. Space Sci.* **277** 181–7
[12] Cranmer S R 2001 *J. Geophys. Res.* **106** 937–24
[13] Isenberg P A 2001 *Space Sci. Rev.* **95** 119–31
[14] Marsch E and Tu C-Y 2001 *J. Geophys. Res.* A **106** 8357–61
[15] Vasenin Y M and Minkova N R 2002 *Proc. 11th Int. Cong. Plasma Physics (AIP Conf. Proc. Series)* Paper 130 at press
[16] Pannekoek A 1922 *Bull. Astron. Inst. Neth., Suppl. Ser.* **1** 118
Rosseland S 1924 *Mon. Not. R. Astron. Soc.* **84** 728
[17] Lemaire J and Scherer M 1971 *J. Geophys. Res.* **76** 7479–90
[18] Koehnlein W 1996 *Solar Phys.* **169** 209–13
[19] Rubtsov S N, Yakovlev O I and Efimov A I 1987 *Space Res.* **25** pp 620–5
[20] Yakubov V P 1997 *Doppler Superlargebase Interferometery* (Tomsk: Vodoley) p 136